Specific heat capacities of gases (Cp & Cv) and Mayer's relation

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Specific heat capacities of gases

Heat capacity : If a material of mass m absorbs heat Q, its temperature rises through ΔT

$$\text{Heat capacity} = \frac{Q}{\Delta T}$$

Specific heat (C) : Heat capacity per unit mass

$$\therefore Specific\ heat = \frac{Heat\ capacity}{mass}$$

$$C = \frac{Q}{m \cdot \Delta T} \qquad \text{(or)} \qquad C = \frac{Q}{m \cdot\ \theta}$$

Definition : the specific heat of material is defined as the quantity of heat required to raise temperature of unit mass of the material through 1 degree.

Unit: In C.G.S calories per gram per ^{o}C. In S.I. it is Joules per Kg per ^{o}C.

For example :

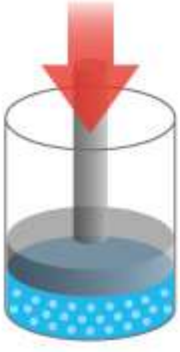
Specific heat of water = 1 cal/gm ^{o}C

= 1 Kilo calories/Kg ^{o}C

= 4.18×10^3 Joules/Kg ^{o}C

The gases are compressible, the specific heat of gas may vary from zero to infinity

For example :



If a gas is compressed, its temperature rises without supplying any heat to it i.e. Q=0

Hence, Specific heat $C = \frac{Q}{m \cdot \Delta T} = 0$

If the is allowed to expand freely, without any rise in temperature (ΔT=0) then

$$C = \frac{Q}{m \cdot 0} = \infty$$

The gas has two specific heats

(i). C_p, the specific heat at constant pressure
(ii) C_v, the specific heat at constant volume

Specific heat at constant pressure (C_P) :

It is defined as the amount of heat required to raise the temperature of unit mass of a gas through 1 ^oC, when its pressure is kept constant.

$$C_p = \left[\frac{\Delta Q}{\Delta T}\right]_p$$

Specific heat at constant volume (C_v) :

It is defined as the amount of heat required to raise the temperature of unit mass of a gas through 1 ^oC, when its volume is kept constant.

$$C_v = \left[\frac{\Delta Q}{\Delta T}\right]_v$$

$\boldsymbol{C_p}$ ***is greater than*** $\boldsymbol{C_v}$ ($\boldsymbol{C_p} > \boldsymbol{C_v}$):

The heat is supplied to a gas and is allowed to expand at constant pressure. Then

(i). It raises the temperature of the gas (i.e. increase in its internal energy)

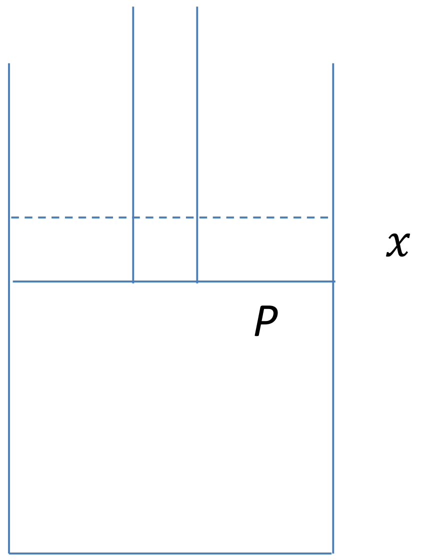
(ii). It does work in expanding the gas against the external pressure

$$\begin{aligned}\delta Q &= dU + \delta W \\ &= dU + PdV\end{aligned}$$

When gas is heated at constant volume, no work is done ($\delta W = PdV = 0$) and hence whole of the heat supplied is used to raise its temperature. Thus more heat is required for increasing the temperature of the gas through 1 ^{o}C at constant pressure than at constant volume. Hence $\boldsymbol{C_p} > \boldsymbol{C_v}$.

Relation between C_p and C_v :

Consider one gram of a gas at a pressure *P*, volume *V* and temperature *T*. Heat is supplied to the gas to raise its Temperature through *dT*. As pressure has to rain constant,



Work done, *W* = $P \times A \times x = P \times dV$

Where dV is the change in volume

From the gas equation

$$PV = rT$$

Differentiating,

$$PdV + VdP = rdT$$

But *dP=0*

$$PdV = rdT$$

Work done in heat units= $\frac{r\cdot dT}{J}$ calories

Heat supplied $dQ = 1\times\ C_p \times dT$

Molar specific heat

$dU = 1\times\ C_v \times dT$

From first law of thermodynamics

$$dQ = dU + dW$$

$$1\times\ C_p \times dT = 1\times\ C_v \times dT + \frac{r\cdot dT}{J}$$

$$C_p =\ C_v + \frac{r}{J}$$

$$\boxed{C_v - C_p = \frac{r}{J}}$$

Where r is the gas constant for one gram of a gas. If C_p and C_v represent gram molecular specific heats, then

$$\boxed{C_v - C_p = \frac{R}{J}}$$

Where R is the universal gas constant and This expression is called **Mayer's relation**